

DFTT-8/2008

# A comment on discrete Kalb-Ramond field on orientifold and rank reduction.

**I. Pesando**

*Dipartimento di Fisica Teorica, Università di Torino and INFN, Sezione di Torino,  
via P. Giuria 1, I-10125, Torino, Italy*

## **Abstract**

We show that the rank reduction of the gauge group on orientifolds in presence of non vanishing discrete Kalb-Ramond field can be explained by the presence of an induced field strength in a non trivial bundle on the branes. This field strength is also necessary for the tadpole cancellation and the number of branes is left unchanged by the presence of the discrete Kalb-Ramond background.

# 1 Introduction.

Since its discovery in ([1]) the phenomenon of rank reduction of gauge group in presence of a non vanishing  $B$  in an orientifold has been revisited a certain number of times ( see for example ([2], [3], [4]), [5]). We will show that there are still some points which are worth understanding; in particular we will show that the rank reduction can be understood as an effect of an induced non trivial gauge bundle on the compact part of branes.

Even in presence of  $B_{ij}$  the usual way used to describe the condition of no momentum flow through the ends of the open string is  $p_{Li} = -p_{Ri}$  ([6],[4] and [2]), this can be better rewritten as

$$(p_{Li} + p_{Ri}) |B\rangle = (n_i - B_{ij}m^j) |B\rangle = 0 \quad (1)$$

with  $|B\rangle$  the boundary state. On the other side the reflection condition for a generic constant background on a torus can be written as (see for example ([7], [8], [9]))

$$\left( n_i - \hat{F}_{ij}m^j \right) |B(E, F)\rangle = 0 \quad (2)$$

where  $\hat{F}_{ij} = 2\pi\alpha'qF_{ij}$  is the adimensional field strength. Comparing these two equations (1) and (2) we find

$$\hat{F}_{ij} = B_{ij} \quad (3)$$

This equality is partially naive because the interesting values of  $B$  are fractional and therefore the field strength cannot be the field strength of a trivial  $U(1)$  bundle on a torus. The previous equation (3) is naive because does not seem to consider the effect of the worldsheet parity  $\Omega$  on the stacks of branes where we need to define the bundle. Anyhow it is a clear hint that something is missing.

To make this equality more precise we exam once again the different amplitudes involved in the orientifold projection paying particular attention to the dependence on the metric, either open or closed. This will essentially confirm eq. (3), more precisely we find a field strength

$$2\pi\alpha'qF = \frac{1}{2}B_{ij}dx^i dx^j \mathbb{I}_{2^{D/2}} \quad (4)$$

of a  $SO(2^{D/2})$  bundle along with the proper transition functions (as in eq. (35) when we write  $\mathbb{I}_{2^{D/2}} = \mathbb{I}_{2^{D/2-1}} \oplus \mathbb{I}_{2^{D/2-1}}$ ). This turns out not to be the only new ingredient: in order to be able to cancel the tadpole we discover that we need some extra signs due to the (momentum dependent) Chan-Paton matrices which naturally emerge in the description of the string on a non trivial bundle ([7]). Moreover in order to get a perfect match of the open and closed string computations we find that a phase quadratic in winding is necessary in the definition of the crosscap state. The non trivial bundle then leads naturally to rank reduction because of a mechanism à la Scherk-Schwarz.

The paper is organized as follows. In section 2 we fix our conventions and review the description of an open string in a non trivial bundle on a torus and we discuss how the effect of a non trivial bundle can be interpreted to give a reduction à la Scherk-Schwarz. Then in section 3 we discuss the action of the worldsheet parity  $\Omega$  on the states in a non trivial bundle. In section 4 we perform the usual Klein bottle computation paying attention to the dependence on the metric of the result and we state the final form of the crosscap, momentum dependent signs included. In sections 5 and 6 we perform the open string computation and we derive the results (almost) correctly guessed in the previous literature. Finally in section 7 we draw our conclusions.

## 2 A short review of open string on non trivial bundles.

In the following we use the notations used in ([7], [8]). In particular the closed string Hamiltonian in a metric background  $E_{ij} = G_{ij} + B_{ij}$  on a generic flat space  $R^{D-d} \otimes T^d$  ( $D = 26$ ) can be written as follows:

$$\begin{aligned} \frac{H_c - 4}{2} &= L_0 + \tilde{L}_0 = \frac{\alpha'}{4\pi} \int_0^\pi d\sigma [P_L^2 + P_R^2], \\ &= N + \tilde{N} + \frac{1}{2} [G_{ij} \hat{m}^i \hat{m}^j + (\hat{n}_i - B_{ik} \hat{m}^k) G^{ij} (\hat{n}_j - B_{jh} \hat{m}^h)] + \frac{\alpha'}{2} G^{\mu\nu} k_\mu k_\nu \end{aligned} \quad (5)$$

where  $i, j, \dots = 1, 2, \dots d$  ;  $\mu, \nu = 0, d+1, \dots D$  and the explicit expressions of  $L_0$  and  $\tilde{L}_0$  are given by

$$\begin{aligned} L_0 &= \frac{\alpha'}{4} G^{\mu\nu} k_\mu k_\nu + \frac{\alpha'}{4} G_{ij} p_R^i p_R^j + N \quad ; \quad N = \sum_{n=1}^{\infty} G_{\mu\nu} \alpha_{-n}^\mu \alpha_n^\nu + G_{ij} \alpha_{-n}^i \alpha_n^j \\ \tilde{L}_0 &= \frac{\alpha'}{4} G^{\mu\nu} k_\mu k_\nu + \frac{\alpha'}{4} G_{ij} p_L^i p_L^j + \tilde{N} \quad ; \quad \tilde{N} = \sum_{n=1}^{\infty} G_{\mu\nu} \tilde{\alpha}_{-n}^\mu \tilde{\alpha}_n^\nu + G_{ij} \tilde{\alpha}_{-n}^i \tilde{\alpha}_n^j, \end{aligned} \quad (6)$$

and the spectrum of the compact momenta by

$$(k_i) \begin{pmatrix} L \\ R \end{pmatrix} = G_{ij} (p^j) \begin{pmatrix} L \\ R \end{pmatrix} = \frac{1}{\sqrt{\alpha'}} [(n_i - B_{ij} m^j) \pm G_{ij} m^j], \quad (7)$$

The non vanishing commutators are

$$[x_L^i, p_L^j] = i G^{i,j} \quad , \quad [\alpha_{Ln}^i, \alpha_{Lm}^j] = n \delta_{n+m,0} G^{i,j} \quad (8)$$

and similarly for the right movers and for the non compact directions. The normalization of the zero modes is

$$\langle k_\mu, n_i, m^i | k'_\mu, n'_i, m^{i'} \rangle = (2\pi)^{D-d} \delta^{D-d} (k_\mu - k'_\mu) (2\pi \sqrt{\alpha'})^d \delta_{n,n'} \delta_{m,m'} . \quad (9)$$

Let us now consider the open string in a metric background given by  $E_{ij}$  and in presence of a constant background field  $F_{ij}$ . We assume that this background field is

$$\hat{F}_{2a-1,2a} = 2\pi\alpha' q F_{2a-1,2a} = \frac{f_a}{L_{(a)}} \mathbb{I}_{\prod_{b=1}^r L_{(b)} N_1} , \quad a = 1, \dots, r \quad (10)$$

with all the other components vanishing, i.e. the rank of the field strength is  $r$ . This background field is obtained from the gauge field (up to gauge choices)

$$A_i = \frac{1}{2} F_{ji} x^j \mathbb{I}_{\prod_{b=1}^r L_{(b)} N_1} \quad (11)$$

along with the transition functions which we take to be

$$\begin{aligned} \Omega_1 &= e^{i2\pi\theta_1} e^{-i\frac{f_1}{L_{(1)}} \frac{x^2}{2\sqrt{\alpha'}}} Q_{L_1} \otimes \mathbb{I}_{L_2} \dots \mathbb{I}_{N_1} , & \Omega_2 &= e^{i2\pi\theta_2} e^{i\frac{f_1}{L_{(1)}} \frac{x^1}{2\sqrt{\alpha'}}} P_{L_1}^{-f_1} \otimes \mathbb{I}_{L_2} \dots \mathbb{I}_{N_1} \\ \Omega_3 &= e^{i2\pi\theta_3} \mathbb{I}_{L_1} \otimes e^{-i\frac{f_2}{L_{(2)}} \frac{x^4}{2\sqrt{\alpha'}}} Q_{L_2} \dots \mathbb{I}_{N_1} , & \Omega_4 &= e^{i2\pi\theta_4} \mathbb{I}_{L_1} \otimes e^{i\frac{f_2}{L_{(2)}} \frac{x^3}{2\sqrt{\alpha'}}} P_{L_2}^{-f_2} \dots \mathbb{I}_{N_1} \\ &\vdots & & \\ \Omega_{2r+1} &= e^{i2\pi\theta_{2r+1}} \mathbb{I}_{L_1} \otimes \mathbb{I}_{L_2} \dots \mathbb{I}_{N_1} , \dots & \Omega_d &= e^{i2\pi\theta_d} \mathbb{I}_{L_1} \otimes \mathbb{I}_{L_2} \dots \mathbb{I}_{N_1} \end{aligned} \quad (12)$$

where  $e^{i2\pi\theta_i}$  are the abelian Wilson lines. The block diagonal field strength (10), the fact we are working on a torus  $T^d$  and the cocycle conditions on the transition functions, i.e  $\Omega_j(x^k + 2\pi\sqrt{\alpha'}\delta_i^k)\Omega_i(x^k) = \Omega_i(x^k + 2\pi\sqrt{\alpha'}\delta_j^k)\Omega_j(x^k)$ , oblige to consider the gauge group to be factorized as  $\otimes_{a=1}^r U(L_{(a)}) \otimes U(N_1)$ : this does not happen on a non compact surface and it is responsible for the rank reduction as we can see from the physical states in eq. (19) and we discuss after eq. (24).

On this background the dipole string Hamiltonian is then given by

$$\begin{aligned} H_o - 1 = L_0 &= \alpha' G^{\mu\nu} k_\mu k_\nu + \alpha' p^i \mathcal{G}_{ij} p^j + \sum_{n=1}^{\infty} n G_{\mu\nu} \alpha_n^{\dagger\mu} \alpha_n^\nu + \mathcal{G}_{ij} a_n^{\dagger i} a_n^j \\ &= \alpha' G^{\mu\nu} k_\mu k_\nu + \mathcal{G}^{ij} \frac{n_i}{L_i} \frac{n_j}{L_j} + \sum_{n=1}^{\infty} G_{\mu\nu} \alpha_n^{\dagger\mu} \alpha_n^\nu + \mathcal{G}_{ij} a_n^{\dagger i} a_n^j \end{aligned} \quad (13)$$

with the open string metric given by

$$\mathcal{G}_{ij} = G_{ij} - \mathcal{B}_{ik} G^{kh} \mathcal{B}_{hj} = \mathcal{E}_{ik}^T G^{kh} \mathcal{E}_{hj} \quad (14)$$

In the previous expression we have defined the following quantities

$$\mathcal{B}_{ij} = B_{ij} - \hat{F}_{ij} \quad (15)$$

$$\mathcal{E}_{ij} = G_{ij} - \mathcal{B}_{ij} = G_{ij} - B_{ij} + \hat{F}_{ij} \quad (16)$$

The compact momenta have spectrum

$$p^i = \mathcal{G}^{ij} \frac{1}{\sqrt{\alpha'}} \frac{n_i}{L_i} \quad (17)$$

where we have defined  $L_{2a} = L_{2a-1} = L_{(a)}$  for  $1 \leq a \leq r$ ,  $L_i = 1$  for  $2r < i \leq d$ . The non-vanishing commutation relations in compact directions are:

$$[x^i, p^j] = i\mathcal{G}^{ij} ; \quad [x^i, x^j] = i 2\pi\alpha'\Theta^{ij} ; \quad [\alpha_n^i, \alpha_m^j] = n\delta_{n+m,0}\mathcal{G}^{ij} \quad (18)$$

where  $\mathcal{E}^{-1} = \mathcal{G}^{-1} - \Theta$ . On this background the normalized string states are given by

$$\begin{aligned} |\chi; k_\mu, n_i; u\rangle &= \frac{1}{(2\pi\sqrt{\alpha'})^{d/2}} |\chi\rangle \otimes |k_\mu\rangle \otimes \Lambda_{L_{(1)}; I_1 J_1}(n_1, n_2) |\frac{n_1}{\sqrt{\alpha'} L_{(1)}}, \frac{n_2}{\sqrt{\alpha'} L_{(1)}}\rangle_p |J_1 I_1\rangle \\ &\otimes \Lambda_{L_{(2)}; I_2 J_2}(n_3, n_4) |\frac{n_3}{\sqrt{\alpha'} L_{(2)}}, \frac{n_4}{\sqrt{\alpha'} L_{(2)}}\rangle_p |J_2 I_2\rangle \dots \\ &\otimes T_{u_{N_1; I_{r+1} J_{r+1}}} |\frac{n_{2r+1}}{\sqrt{\alpha'}}, \dots, \frac{n_d}{\sqrt{\alpha'}}\rangle |J_{r+1} I_{r+1}\rangle \end{aligned} \quad (19)$$

where  $|\chi\rangle$  is the collective name for the quantum numbers associated with the non zero modes,  $|\frac{n_1}{\sqrt{\alpha'} L_{(1)}}, \frac{n_2}{\sqrt{\alpha'} L_{(1)}}\rangle_p$  is a momentum eigenvector,  $|J_1 I_1\rangle$  is an element of basis for the color indexes (see ([10]) for more details). The meaning of writing  $\Lambda_{L_{(1)}; I_1 J_1}(n_1, n_2) |J_1 I_1\rangle$  is that for a given momentum  $\left(\frac{n_1}{\sqrt{\alpha'} L_{(1)}}, \frac{n_2}{\sqrt{\alpha'} L_{(1)}}\right)$  not all the possible  $L_{(1)}^2$   $|J_1 I_1\rangle$  color index combinations are possible, as it is usual with the trivial bundle, but only one.

In the eq. (19)  $T_u$  are the usual  $N_1^2$  hermitian  $u(N_1)$  generators and can be traded for the  $N_1^2$  color states  $|J_{r+1} I_{r+1}\rangle$ .

The  $\Lambda$  are the hermitian momentum dependent Chan-Paton matrices given by

$$\Lambda_{L; IJ}(n_1, n_2) = \frac{1}{\sqrt{L}} e^{-i\frac{\pi}{L}\hat{h}n_1 n_2} \left( Q_L^{\hat{h}n_2} P_L^{-n_1} \right)_{IJ} , \quad 0 \leq I, J < L \quad (20)$$

with  $\hat{h}f \equiv -1 \bmod L$  which enjoy the hermitian conjugation property

$$\Lambda_L^\dagger(n_1, n_2) = \Lambda_L(-n_1, -n_2) \quad (21)$$

and are normalized as

$$\text{tr} \left( \Lambda_L^\dagger(n_1, n_2) \Lambda_L(m_1, m_2) \right) = \delta_{n,m} \quad (22)$$

In particular for the  $L = 2$  case, which is of our interest, the explicit form of the  $\Lambda_2$  matrices is

$$\Lambda_{2; IJ}(n_1, n_2) = \frac{1}{\sqrt{2}} e^{-i\frac{\pi}{2}n_1 n_2} \left( \sigma_3^{n_2} \sigma_1^{-n_1} \right)_{IJ} , \quad 0 \leq I, J < 2 \quad (23)$$

To show that the states in eq. (19) are normalized we perform a computation like

$$\begin{aligned} &\langle MK|_p \langle \frac{n_1}{\sqrt{\alpha'} L}, \frac{n_2}{\sqrt{\alpha'} L} | \left( \Lambda_L^\dagger \right)_{KM} (n_1, n_2) \Lambda_{L; IJ}(m_1, m_2) | \frac{m_1}{\sqrt{\alpha'} L}, \frac{m_2}{\sqrt{\alpha'} L} \rangle_p |JI\rangle \\ &= (2\pi\sqrt{\alpha'})^2 \delta_{n,m} \delta_{K,I} \delta_{M,J} \left( \Lambda_L^\dagger \right)_{KM} (n_1, n_2) \Lambda_{L; IJ}(m_1, m_2) \\ &= (2\pi\sqrt{\alpha'})^2 \delta_{n,m} \text{tr} \left( \Lambda_L^\dagger(n_1, n_2) \Lambda_L(m_1, m_2) \right) = (2\pi\sqrt{\alpha'})^2 \delta_{n,m} \end{aligned} \quad (24)$$

Finally we notice that even if we start with  $\prod_{b=1}^r L_{(b)} N_1$  branes the number of massless states  $k_\mu^2 = 0$  is only  $N_1^2$ . This does not mean that we have not  $(\prod_{b=1}^r L_{(b)} N_1)^2$  states as we naively would expect but that some of them become massive, with a mass of order  $\frac{1}{L}$ : it is essentially a Scherk-Schwarz reduction mechanism and is the key idea of the explanation of the rank reduction.

For later convenience and use in the computation of the annulus and the Moebius amplitude we write the spectral decomposition of the unity as

$$\begin{aligned} \mathbb{I} &= \int \frac{d^{D-d} k_\mu}{(2\pi)^{D-d}} \sum_{\chi, n_i, u} |\chi; k_\mu, n_i; u\rangle \langle \chi; k_\mu, n_i; u| \\ &= \int \frac{d^{D-d} k_\mu}{(2\pi)^{D-d}} \sum_{\chi, n_i, u} |\chi\rangle \otimes |k_\mu\rangle \otimes \Lambda_{L_{(1)}; I_1 I_1}(n_1, n_2) \left| \frac{n_1}{\sqrt{\alpha' L_{(1)}}}, \frac{n_2}{\sqrt{\alpha' L_{(1)}}} \right\rangle_p |J_1 I_1\rangle \\ &\quad \frac{1}{(2\pi\sqrt{\alpha'})^d} \dots \langle M_1 K_1|_p \left\langle \frac{n_1}{\sqrt{\alpha' L_{(1)}}}, \frac{n_2}{\sqrt{\alpha' L_{(1)}}} \right| \Lambda_{L_{(1)}; K_1 M_1}^\dagger(n_1, n_2) \otimes \langle k_\mu| \otimes \langle \chi| \end{aligned} \quad (25)$$

which can be used to define the trace as

$$Tr(O) = \int \frac{d^{D-d} k_\mu}{(2\pi)^{D-d}} \sum_{\chi, n_i, u} \langle \chi; k_\mu, n_i; u| O | \chi; k_\mu, n_i; u \rangle \quad (26)$$

### 3 The action of $\Omega$ on a non trivial bundle.

Before computing the open amplitudes we must discuss the action of the  $\Omega$  operator. Our picture is to start with a stack of  $\prod_{b=1}^r L_{(b)} N_1$  branes which gets mapped to a second image stack of  $\prod_{b=1}^r L_{(b)} N_1$  branes by the  $\Omega$  action therefore the generic element for the color basis will be

$$|CD\rangle = \begin{pmatrix} |cd\rangle & |cd'\rangle \\ |c'd\rangle & |c'd'\rangle \end{pmatrix} \quad (27)$$

where we have used the usual convention of using the prime to denote the color indexes of the mirror stack and the color index  $c$  is the collective name of the indexes  $(I_1 I_2 \dots I_{r+1})$ .

On the fluctuations around the trivial background we use the usual action

$$\Omega |\chi, k_i = \frac{n_i + \theta_{Ci} - \theta_{Di}}{\sqrt{\alpha'}}; CD\rangle = (\gamma_\Omega)_{CD_1} |\chi^\Omega, k_i = \frac{n_i + \theta_{C_1 i} - \theta_{D_1 i}}{\sqrt{\alpha'}}; C_1 D_1\rangle (\gamma_\Omega^{-1})_{C_1 D} \quad (28)$$

where  $C, D, \dots = 1, \dots, 2 \prod_{b=1}^r L_{(b)} N_1$ ,  $\theta_C$  is the Wilson line on the  $C$ -th brane and we assume that  $\gamma_\Omega$  have the simplest off diagonal form

$$\gamma_\Omega = \begin{pmatrix} & \mathbb{I} \\ \mathbb{I} & \end{pmatrix} \quad (29)$$

which is the best suited form to describe the picture where we start with a stack of  $\prod_{b=1}^r L_{(b)} N_1$  branes which get mapped to a new stack. The  $\Omega$  acts on the non zero modes as

$$\Omega \alpha_n \Omega = (-)^n \alpha_n \quad (30)$$

Therefore the gluons which survive the projection are those whose Chan Paton matrices  $\Lambda$  satisfy  $\gamma_\Omega \Lambda \gamma_\Omega^{-1} = -\Lambda$  which with our choice of  $\gamma_\Omega$  reads<sup>1</sup>

$$\Lambda = \begin{pmatrix} H & A \\ A^\dagger & -H^T \end{pmatrix}, \quad A^T = -A, \quad H^\dagger = H \quad (31)$$

This form of an element of the algebra suggests that if we start with the transition functions  $\Omega_{(1)i}$  on the original stack the transition functions for the two stacks are given by

$$\Omega_i = \begin{pmatrix} \Omega_{(1)i} & \\ & \Omega_{(1)i}^* \end{pmatrix} \quad (32)$$

This form is consistent with the naive expectation that if the first stack has common Wilson lines described by

$$\Omega_{(1)i} = e^{i2\pi\theta_i} \mathbb{I} \quad (33)$$

the Wilson lines on the image stack are opposite so that the strings which connect the two stacks have double Wilson line  $\pm 2\theta_i$  which corresponds to halving the T-dual torus.

If we now consider a non trivial bundle of the kind described in the previous section when  $B = 0$  we can still take the transition functions as in eq. (32). Since in the transition functions it is essentially encoded the background field, see eq. (12), we deduce that the background field strength on the image stack is  $F^\Omega = -F$  because the transition functions on the image stack are the complex conjugate of the original ones. This is nice since  $\mathcal{E}^\Omega = \mathcal{E}^T$  and therefore  $\mathcal{G}^\Omega = \mathcal{G}$ .

The action of  $\Omega$  on the fluctuation around this non trivial background is still in nuce given by eq. (28 )

$$\Omega |\chi; CD\rangle = (\gamma_\Omega)_{CD} |\chi^\Omega; C_1 D_1\rangle (\gamma_\Omega^{-1})_{C_1 D} \quad (34)$$

where we have not written the momenta since the strings connecting the two stacks are discharged ones.

If we now consider a non trivial gauge background with some half integer  $B$  turned on together with the the transition functions as in eq. (32) then we lose the nice properties  $\mathcal{E}^\Omega = \mathcal{E}^T$  and  $\mathcal{G}^\Omega = \mathcal{G}$ . On the other hand, as we discuss later in

---

<sup>1</sup> To obtain the  $so(2 \prod_{b=1}^r L_{(b)} N_1)$  matrices we have to choose  $\gamma'_\Omega = \begin{pmatrix} \mathbb{I} & \\ & \mathbb{I} \end{pmatrix}$ , choice which is anyhow equivalent to  $\gamma_\Omega$  since  $\gamma'_\Omega = -iU^\dagger \gamma_\Omega U^*$  where  $U = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbb{I} & i\mathbb{I} \\ i\mathbb{I} & \mathbb{I} \end{pmatrix}$ .

section 5, tadpole cancellation requires that we freeze  $\hat{F} - B = \hat{F}^\Omega - B = 0$ , i.e.  $\hat{F}^\Omega = -\hat{F} + 2B$ . This requires that we consider the transition functions

$$\Omega_i = \begin{pmatrix} \Omega_{(1)i} & \\ & e^{-2iB_{ij}\frac{x^j}{2\sqrt{\alpha'}}\Omega_{(1)i}^*} \end{pmatrix} = \begin{pmatrix} \Omega_{(1)i} & \\ & \Omega_{(1)i} \end{pmatrix} \quad (35)$$

and this in turn implies that on both stacks we have the same field strength so that also the strings connecting the two stacks are dipole ones therefore the action of  $\Omega$  on the fluctuation around this non trivial background is given by

$$\Omega|\chi, k_i = \frac{\frac{n_i}{L_i} + \theta_{Ci} - \theta_{Di}}{\sqrt{\alpha'}}; CD\rangle = (\gamma_\Omega)_{CD_1}|\chi^\Omega, k_i = \frac{\frac{n_i}{L_i} + \theta_{C_1i} - \theta_{D_1i}}{\sqrt{\alpha'}}; C_1D_1\rangle(\gamma_\Omega^{-1})_{C_1D} \quad (36)$$

As a consequence the spectrum is essentially given by four times the spectrum in eq. (19); using the the same notation as in eq. (27) and in presence of the Wilson lines (33) we have:

$$\begin{pmatrix} |\chi; k_\mu, n_i; u\rangle & |\chi; k_\mu, n_i + 2L_i\theta_i; u\rangle \\ |\chi; k_\mu, n_i - 2L_i\theta_i; u\rangle & |\chi; k_\mu, n_i; u'\rangle \end{pmatrix} \quad (37)$$

## 4 The Klein bottle.

We start to compute the Klein bottle<sup>2</sup> for a space time  $R^{D-d} \otimes T^d$  ( $D = 26$ )

$$Z_K = \int \frac{d\tau}{\tau} \text{Tr}_c \left( \frac{\Omega}{2} e^{-\pi\tau H_c} \right) \quad (38)$$

where we have used  $\tau$  as integration variable since this amplitude must be interpreted as open channel amplitude even if derived it projecting a closed string one with the insertion of  $\frac{\Omega}{2}$  to implement the orientifold (in this case the worldsheet parity only).

To perform the previous computation we need defining the action of  $\Omega$  ([2]).

Since this is the worldsheet parity which exchange the left and right sector we require that the left momenta lattice  $\{\sqrt{\alpha'} G p_L(n, m) = (n + E^T m)\}$  be equal to the right momenta lattice  $\{\sqrt{\alpha'} G p_R(n, m) = (n - Em)\}$ . This means that for any  $n, m$  we can find some  $n', m'$  so that  $p_L(n, m) = p_R(n', m')$ , if we want this relation be valid for any metric  $G$  we find  $m' = -m$  and  $n' = n - 2Bm$ . This last relation must be true for all  $m$  and  $n$  and therefore we restrict  $2B_{ij} \in \mathbb{Z}$ . Hence we define the action of  $\Omega$  as

$$\begin{aligned} \Omega|n, m\rangle &= |n - 2Bm, -m\rangle \\ \Omega\alpha_n\Omega &= (-1)^n \tilde{\alpha}_n \end{aligned} \quad (39)$$

<sup>2</sup> We use the amplitude normalizations given in the review ([10]), from which we take also the following notations and relations:  $f_1(q) \equiv q^{\frac{1}{12}} \prod_{n=1}^{\infty} (1 - q^{2n}) = e^{-\pi\tau/12} \prod_{n=1}^{\infty} (1 - e^{-2n\pi\tau})$  with  $q = e^{-\pi\tau} \in R$ ,  $f_1(e^{-\pi t}) = \frac{1}{\sqrt{t}} f_1(e^{-\frac{\pi}{t}})$  and  $f_1(ie^{-\pi t}) = \frac{1}{\sqrt{2t}} f_1(ie^{-\frac{\pi}{4t}})$

Without loss of generality we can assume  $B$  to be block diagonal in the space indexes.

The Klein bottle amplitude is then given by

$$\begin{aligned} Z_K &= \int \frac{d\tau}{2\tau} \int \frac{d^{D-d}k^\mu}{(2\pi)^{D-d}} (2\pi)^{D-d} \delta^{D-d}(k^\mu - k^\mu) e^{-\pi\tau\alpha'(k^\mu)^2} \sum_{n \in Z^d} e^{-\pi\tau n^T G^{-1} n} \frac{e^{4\pi\tau}}{(\prod (1 - e^{-4\pi\tau n}))^{D-2}} \\ &= \int \frac{d\tau}{2\tau} \frac{V_{nc}}{(\alpha'\tau)^{\frac{D-d}{2}}} \sum_{n \in Z^d} e^{-\pi\tau n^T G^{-1} n} \frac{e^{-\pi\tau \frac{26-D}{6}}}{(f_1(e^{-2\pi\tau}))^{D-2}} \end{aligned} \quad (40)$$

where the power 4 in  $e^{-4\pi\tau n}$  is due to the left/right identification and  $V_{nc} = (2\pi)^{D-d} \delta^{D-d}(0)$ . We can now perform the modular transformation on the  $f_1$  and the Poisson resummation to get

$$\begin{aligned} Z_K &= \frac{V_{nc}}{\sqrt{\alpha'}^{D-d}} \int \frac{d\tau}{4\tau} \frac{1}{\tau^{\frac{D-d}{2}}} (\det \tau G^{-1})^{-1/2} \sum_{u \in Z^d} e^{-\pi \frac{1}{\tau} u^T G u} \frac{(2\tau)^{\frac{D-2}{2}} e^{-\pi\tau \frac{26-D}{6}}}{(f_1(e^{-\frac{\pi}{2\tau}}))^{D-2}} \\ &= \frac{V_{nc}}{\sqrt{\alpha'}^{D-d}} \sqrt{\det G} 2^{\frac{D}{2}} \int \frac{d\tau}{4\tau^2} \sum_{u \in Z^d} e^{-\pi \frac{1}{\tau} u^T G u} \frac{e^{-\pi\tau \frac{26-D}{6}}}{(f_1(e^{-\frac{\pi}{2\tau}}))^{D-2}} \end{aligned} \quad (41)$$

Renaming  $t = \frac{1}{2\tau}$  we can finally write in the closed channel

$$Z_K = 2^{\frac{D}{2}} \frac{V_{nc}}{2\alpha'^{\frac{D-d}{2}}} \sqrt{\det G} \int dt \sum_{u \in Z^d} e^{-\pi 2t u^T G u} \frac{e^{-\pi \frac{1}{t} \frac{26-D}{6}}}{(f_1(e^{-\pi t}))^{D-2}} \quad (42)$$

where it is important to stress that the amplitude is proportional to the determinant of the closed string metric  $G$ , fact that is reflected in the normalization of the crosscap<sup>3</sup>:

$$\begin{aligned} |C(E)\rangle &= \frac{T'_{25}}{2} 2^{\frac{D}{4}} (\det G)^{\frac{1}{4}} |C(E)\rangle_{zm} |C(E)\rangle_{nzm} \\ |C(E)\rangle_{zm} &= |k_\mu = 0\rangle \otimes \sum_{s \in Z^d} e^{i\frac{\pi}{2} \sum_{i < j} m^i B_{ij} m^j} |n_i = B_{ij} m^j, m^i = 2s^i\rangle \\ |C(E)\rangle_{nzm} &= e^{-\sum_{n=1}^{\infty} (-)^n a_n^{\mu\dagger} G_{\mu\nu} \tilde{a}_n^{\nu\dagger} - \sum_{n=1}^{\infty} (-)^n a_n^{i\dagger} G_{ij} \tilde{a}_n^{j\dagger}} |0, \tilde{0}\rangle \end{aligned} \quad (43)$$

which can be obtained from the closed string computation

$$Z_K = \langle C(E) | \frac{\alpha'\pi}{2} \int_0^\infty dl e^{-\pi l(L_0 + \tilde{L}_0 - 2)} \delta_{L_0, \tilde{L}_0} |C(E)\rangle \quad (44)$$

up to the phase  $e^{i\frac{\pi}{2} \sum_{i < j} m^i B_{ij} m^j} = e^{i\pi \sum_{i < j} s^i 2B_{ij} s^j}$  that can be determined from the interference term with the boundary state, the Moebius amplitude. We notice that this phase is half the corresponding one of the boundary state (49).

---

<sup>3</sup> The dependence on  $G$  and not on  $E^T E^{-1} G$  in the non zero mode part and the non trivial phase  $e^{i2\pi \sum_{i < j} s^i B_{ij} s^j}$  in the zero mode part can be seen from the Moebius amplitude.

## 5 The annulus amplitude.

Now we want to compute annulus amplitude for open strings associated with  $2 \prod_{b=1}^r L_{(b)} N_1$  branes with the gauge bundles described in section 3<sup>4</sup>

$$Z_A = 2 * \int \frac{d\tau}{2\tau} \text{Tr}_o \left( \frac{1}{2} e^{-2\pi\tau H_o} \right) \quad (45)$$

where the trace is taken also over the Chan Paton factors and the factor 2 takes into account the two possible orientations. Moreover the factor  $\frac{1}{2}$  has been inserted into the trace as the part of the projector  $\frac{1+\Omega}{2}$  which contributes to the annulus.

Having defined the matrix  $L = \text{diag}(L_i)$  we get therefore

$$\begin{aligned} Z_A &= N^2 \int \frac{d\tau}{2\tau} \int \frac{d^{D-d} k^\mu}{(2\pi)^{D-d}} (2\pi)^{D-d} \delta^{D-d}(0) e^{-2\pi\tau\alpha'(k^\mu)^2} \sum_{n \in \mathbb{Z}^d} e^{-2\pi\tau n^T L^{-T} \mathcal{G}^{-1} L^{-1} n} \\ &\quad \times \frac{e^{2\pi\tau}}{\left( \prod (1 - e^{-2\pi\tau n}) \right)^{D-2}} \\ &= N^2 \int \frac{d\tau}{2\tau} \frac{V_{nc}}{(2\alpha'\tau)^{\frac{D-d}{2}}} \sum_{n \in \mathbb{Z}^d} e^{-2\pi\tau n^T L^{-T} \mathcal{G}^{-1} L^{-1} n} \frac{e^{-\pi\tau \frac{26-D}{12}}}{(f_1(e^{-\pi\tau}))^{D-2}} \end{aligned} \quad (46)$$

where  $N^2 = (2N_1)^2$  is the contribution from the Chan-Paton factors. We can now perform the Poisson resummation and modular transformation and we obtain

$$\begin{aligned} Z_A &= N^2 \frac{V_{nc}}{\sqrt{\alpha'^{D-d}}} \int \frac{d\tau}{2\tau} \frac{1}{\tau^{D/2}} (\det 2\tau L^{-T} \mathcal{G}^{-1} L^{-1})^{-1/2} \sum_{u \in \mathbb{Z}^d} e^{-\pi (Lu)^T \frac{\mathcal{G}}{2\tau} (Lu)} \frac{(\tau)^{\frac{D-2}{2}} e^{-\pi\tau \frac{26-D}{12}}}{\left( f_1(e^{-\pi\frac{1}{\tau}}) \right)^{D-2}} \\ &= \frac{N^2 \det L}{2^{\frac{D}{2}}} \frac{V_{nc} \sqrt{\det \mathcal{G}}}{2\alpha'^{\frac{D-d}{2}}} \int \frac{d\tau}{\tau^2} \sum_{u \in \mathbb{Z}^d} e^{-\pi u^T \frac{\mathcal{G}}{2\tau} u} \frac{e^{-\pi\tau \frac{26-D}{12}}}{(f_1(e^{-\frac{\pi}{\tau}}))^{D-2}} \end{aligned} \quad (47)$$

renaming  $t = \frac{1}{\tau}$  we get in the closed string channel

$$Z_A = \frac{N^2 \det L}{2^{\frac{D}{2}}} \frac{V_{nc}}{2\alpha'^{\frac{D-d}{2}}} \sqrt{\det \mathcal{G}} \int dt \sum_{u \in \mathbb{Z}^d} e^{-\pi \frac{t}{2} u^T \mathcal{G} u} \frac{e^{-\pi \frac{2}{t} \frac{26-D}{12}}}{(f_1(e^{-\pi t}))^{D-2}} \quad (48)$$

Here the amplitude is proportional to the determinant of the open string metric  $\mathcal{G}$

---

<sup>4</sup> Here we are cheating a little since we know the answer. The proper computation would be to start with  $\prod_{b=1}^r L_{(b)} N_1$  branes with field strength  $F$  and  $\prod_{b=1}^r L_{(b)} N_1$  branes with field strength  $F^\Omega$ . Generically we would have  $F \neq F^\Omega$  and we would find dicharged string. In any case the boundary state would be given by the sum of two boundary states like (49): one as in eq. (49) with the substitution  $N \rightarrow \frac{N}{2}$  and one obtained from the one in eq. (49) with the substitutions  $N \rightarrow \frac{N}{2}$  and  $F \rightarrow F^\Omega$ . Even in this case we would reach the same conclusion, i.e.  $\hat{F} - B = \hat{F}^\Omega - B = 0$ .

which again can be seen in the normalization of the boundary state

$$\begin{aligned}
|B(E, F)\rangle &= -\frac{T'_{25}}{2} N 2^{-\frac{D}{4}} \sqrt{\det L} (\det \mathcal{G})^{\frac{1}{4}} |B(E, F)\rangle_{zm} |B(E, F)\rangle_{nzm} \\
|B(E, F)\rangle_{zm} &= |k_\mu = 0\rangle \otimes \sum_s e^{i\pi \sum_{i < j} m^i \hat{F}_{ij} m^j} |n_i = \hat{F}_{ij} m^j, m^i = L_i s^i\rangle \\
|B(E, F)\rangle_{nzm} &= e^{-\sum_{n=1}^{\infty} a_n^{\mu\dagger} (G)_{\mu\nu} \tilde{a}_n^{\nu\dagger} - \sum_{n=1}^{\infty} a_n^{i\dagger} (G \mathcal{E}^{-1} \mathcal{E}^T)_{ij} \tilde{a}_n^{j\dagger}} |0, \tilde{0}\rangle
\end{aligned} \tag{49}$$

where we have supposed  $\hat{F}_{ij} \propto \frac{1}{L_i}$  and the a priori non trivial phase in the zero mode part has been derived in ([7],[11]) from the path ordering and it is necessary to allow a correct factorization of the two loop amplitude. Again this phase quadratic in windings cannot be seen from the closed string computation

$$Z_A = \langle B(E, F) | \frac{\alpha' \pi}{2} \int_0^\infty dl e^{-\pi l(L_0 + \tilde{L}_0 - 2)} \delta_{L_0, \tilde{L}_0} |B(E, F)\rangle \tag{50}$$

and actually in the case at hand where  $L = 2$  it is trivial.

Already now it is clear that if we want to cancel the  $|n = 0, m = 0\rangle$  state which is responsible of the tadpole from the sum of the crosscap (43) and the boundary state (49)

$$(|C(E)\rangle + |B(E, F)\rangle)|_{m=n=0} = \frac{T'_{25}}{2} \left( 2^{\frac{D}{4}} (\det G)^{\frac{1}{4}} - N 2^{-\frac{D}{4}} \sqrt{\det L} (\det \mathcal{G})^{\frac{1}{4}} \right) |0, 0\rangle \tag{51}$$

we need to have

$$\det G = \det \mathcal{G} \Rightarrow B + \hat{F} = 0, \tag{52}$$

$$N \sqrt{\det L} = 2^{D/2} \tag{53}$$

Now from the first equation we deduce that  $L = 2$  for each torus where we have a non vanishing  $B \sim \frac{1}{2}$ , therefore we can evaluate  $\det L = 2^{2r}$  where  $r = rk(B)$  is the rank of the matrix  $B$ . It then follows the usual rank reduction of the gauge group.

We can actually generalize eq. (46) to the case of the presence of Wilson lines. We start with a stack of  $\prod_{b=1}^r L_{(b)} N_1 = 2^r N_1$  branes with field strength  $F$  and a common Wilson line  $\theta_1 = \theta$ . The action of  $\Omega$  is to map this stack to an image stack of  $\prod_{b=1}^r L_{(b)} N_2 = 2^r N_2$  ( $N_2 = N_1$ ) branes with  $F_2 = F^\Omega$  and  $\theta_2 = -\theta$ . Therefore we have four different open string sectors which contribute to the annulus:

$$\begin{aligned}
Z_C &= \int \frac{d\tau}{2\tau} \frac{V_{nc}}{(2\alpha'\tau)^{\frac{D-d}{2}}} \left[ (N_1^2 + N_2^2) \sum_{n \in Z^d} e^{-2\pi\tau n^T L^{-1} \mathcal{G}^{-1} L^{-1} n} \right. \\
&\quad \left. + N_1 N_2 \sum_{n \in Z^d} e^{-2\pi\tau(n+2L\theta)^T L^{-1} \mathcal{G}^{-1} L^{-1} (n+2L\theta)} + N_1 N_2 \sum_{n \in Z^d} e^{-2\pi\tau(n-2L\theta)^T L^{-1} \mathcal{G}^{-1} L^{-1} (n-2L\theta)} \right] \\
&\quad \times \frac{e^{-\pi\tau \frac{26-D}{12}}}{(f_1(e^{-\pi\tau}))^{D-2}}
\end{aligned} \tag{54}$$

or in the closed string channel

$$Z_C = \frac{\det L}{2^{\frac{D}{2}}} \frac{V_{nc}}{2\alpha'^{\frac{D-d}{2}}} \sqrt{\det \mathcal{G}} \int dt \frac{e^{-\pi \frac{1}{t} \frac{26-D}{24}}}{(f_1(e^{-\pi t}))^{D-2}} \sum_{u \in \mathbb{Z}^d} e^{-\pi \frac{t}{2} u^T \mathcal{G} u} \left[ (N_1^2 + N_2^2) + N_1 N_2 e^{-4i\pi u^T L \theta} + N_1 N_2 e^{+4i\pi u^T L \theta} \right] \quad (55)$$

from which we can deduce the zero mode part of the boundary to be

$$\begin{aligned} |B(E, F)\rangle_{z.m.} &= |k_\mu = 0\rangle \\ &\otimes \sum_s e^{i\pi \sum_{i < j} m^i \hat{F}_{ij} m^j} \frac{e^{i\pi m^T 2\theta} + e^{-i\pi m^T 2\theta}}{2} |n_i = \hat{F}_{ij} m^j, m^i = L_i s^i\rangle \end{aligned} \quad (56)$$

The presence of Wilson lines does not change the tadpole condition (51).

## 6 The Moebius amplitude.

Let us now check the result on the form of the crosscap and annulus from the explicit computation of the Moebius amplitude while fixing some details.

The Moebius amplitude is given by

$$Z_M = 2 * \int \frac{d\tau}{2\tau} Tr_o \left( \frac{\Omega}{2} e^{-2\pi\tau H_o} \right) \quad (57)$$

where the trace is taken also over the Chan Paton factors and the factor 2 takes into account the two possible orientations. Given the previous case of two stacks we have a contribution only from the strings connecting the two stacks since only these strings are mapped into themselves by the worldsheet parity and we get

$$\begin{aligned} Z_M &= N_1 \int \frac{d\tau}{2\tau} \frac{V_{nc}}{(2\alpha'\tau)^{\frac{D-d}{2}}} \frac{e^{-\pi\tau \frac{26-D}{12}} e^{i\pi \frac{D-2}{24}}}{(f_1(ie^{-\pi\tau}))^{D-2}} \\ &\times \sum_{n \in \mathbb{Z}^d} \left[ e^{-2\pi\tau(n+2L\theta)^T L^{-1} \mathcal{G}^{-1} L^{-1} (n+2L\theta)} + e^{-2\pi\tau(n-2L\theta)^T L^{-1} \mathcal{G}^{-1} L^{-1} (n-2L\theta)} \right] (-)^{n_1 n_2 + \dots + n_{2r-1} n_{2r}} \end{aligned} \quad (58)$$

where  $N_1(-)^{n_1 n_2 + \dots + n_{2r-1} n_{2r}}$  is the contribution from the Chan-Paton factors. In particular the momentum dependent contribution is due to the fact that  $\Lambda_2^T(n_1, n_2) = (-)^{n_1 n_2} \Lambda_2(n_1, n_2)$ . Explicitly for a generic operator  $O(p)$  commuting with  $p$  we have

$$\begin{aligned} &\langle JI' |_p \langle \frac{n_1}{\sqrt{\alpha'L}}, \frac{n_2}{\sqrt{\alpha'L}} | \left( \Lambda_L^\dagger \right)_{I'J} (n_1, n_2) O(p) \Omega \Lambda_{L;I'J}(n_1, n_2) | \frac{n_1}{\sqrt{\alpha'L}}, \frac{n_2}{\sqrt{\alpha'L}} \rangle_p | JI' \rangle \\ &= (2\pi\sqrt{\alpha'})^2 O(n) \left( \Lambda_L^\dagger \right)_{I'J} (n_1, n_2) \Lambda_{L;I'J}(n_1, n_2) \langle JI' | | IJ' \rangle \\ &= (2\pi\sqrt{\alpha'})^2 O(n) \text{tr} \left( \Lambda_L^\dagger (n_1, n_2) \Lambda_L^T (n_1, n_2) \right) = (2\pi\sqrt{\alpha'})^2 O(n) (-)^{n_1 n_2} \end{aligned} \quad (59)$$

where the index  $I$  belongs to the first stack and  $J'$  to the mirror one as the convention we introduced in section 3.

The next step is to perform the Poisson resummation taking care of the extra signs due to the momentum dependent Chan Paton factors. To simplify the explanation of the computation we consider the case of a factorized torus even if the computation works the same way in the non factorized case. For example for the first torus in the factorizes case we find

$$\begin{aligned}
\sum_{(n_1, n_2) \in \mathbb{Z}^2} e^{-2\pi\tau(\frac{n}{2}+2\theta)^T \mathcal{G}_{(1)}^{-1}(\frac{n}{2}+2\theta)} (-)^{n_1 n_2} &= \\
&= \frac{1}{\sqrt{\det 2\tau \mathcal{G}_{(1)}^{-1}}} \sum_{(u^1, u^2) \in \mathbb{Z}^2} e^{-\pi \frac{1}{2\tau} u^T \mathcal{G}_{(1)}^{-1} u} e^{i4\pi u^T \theta} \left[ 1 + e^{i\pi u^1} + e^{i\pi u^2} - e^{i\pi(u^1+u^2)} \right] \\
&= \frac{\sqrt{\det \mathcal{G}_{(1)}}}{2\tau} \sum_{(u^1, u^2) \in \mathbb{Z}^2} e^{-\pi \frac{1}{2\tau} u^T \mathcal{G}_{(1)}^{-1} u} e^{i4\pi u^T \theta} \times 2e^{i\pi u^1 u^2} \tag{60}
\end{aligned}$$

Notice that without the phase  $(-)^{n_1 n_2}$  the terms in the square brackets would give an overall factor of  $4\delta_{u_1, even}\delta_{u_2, even}$  and this would destroy the tadpole cancellation since the Moebius would ahve the wrong normalization.

Given the previous result we can now proceed with the Poisson resummation and the modular transformation to obtain

$$\begin{aligned}
Z_M &= N_1 \frac{V_{nc}}{\sqrt{\alpha'}^{D-d}} \int \frac{d\tau}{2\tau} \frac{1}{(2\tau)^{(D-d)/2}} (\det 2\tau \mathcal{G}^{-1})^{-1/2} \\
&\quad \sum_{u \in \mathbb{Z}^d} e^{-\pi u^T \frac{\mathcal{G}}{2\tau} u} \frac{e^{-4i\pi u^T \theta} + e^{+4i\pi u^T \theta}}{2} 2^r (-)^{u^1 u^2 + \dots + u^{2r-1} u^{2r}} \times \frac{(\tau)^{\frac{D-2}{2}} e^{-\pi\tau \frac{26-D}{12}}}{\left(f_1(e^{-\pi\frac{1}{\tau}})\right)^{D-2}} \\
&= N_1 \sqrt{\det L} \frac{V_{nc} \sqrt{\det \mathcal{G}}}{2\alpha'^{\frac{D-d}{2}}} e^{i\pi \frac{D-2}{24}} \int \frac{d\tau}{\tau^2} \sum_{u \in \mathbb{Z}^d} e^{-\pi u^T \frac{\mathcal{G}}{2\tau} u} (-)^{\sum_{i < j} 2B_{ij} u^i u^j} \frac{e^{-4i\pi u^T \theta} + e^{+4i\pi u^T \theta}}{2} \\
&\quad \times \frac{e^{-\pi\tau \frac{26-D}{12}}}{\left(f_1(ie^{-\frac{\pi}{4\tau}})\right)^{D-2}} \tag{61}
\end{aligned}$$

where we have used  $\det L = 2^{2r}$ . Changing variable to  $t = \frac{1}{4\tau}$  we get finally

$$\begin{aligned}
Z_M &= e^{i\pi \frac{D-2}{24}} N_1 \sqrt{\det L} \frac{V_{nc}}{2\alpha'^{\frac{D-d}{2}}} \sqrt{\det \mathcal{G}} \int dt \sum_{u \in \mathbb{Z}^d} e^{-\pi u^T \frac{\mathcal{G}}{2\tau} u} (-)^{\sum_{i < j} 2B_{ij} u^i u^j} \frac{e^{-4i\pi u^T \theta} + e^{+4i\pi u^T \theta}}{2} \\
&\quad \times \frac{e^{-\pi\tau \frac{26-D}{12}}}{\left(f_1(ie^{-\frac{\pi}{4\tau}})\right)^{D-2}} \tag{62}
\end{aligned}$$

which matches precisely the closed string computation

$$Z_M = \langle C(E) | \frac{\alpha' \pi}{2} \int_0^\infty dl e^{-\pi l(L_0 + \tilde{L}_0 - 2)} \delta_{L_0, \tilde{L}_0} | B(E, F) \rangle . \quad (63)$$

## 7 Conclusions.

We have shown that in order to cancel the tadpole on an orbifold compactification with a non trivial discrete half integer  $B_{ij}$  turned on it is necessary to turn an equal constant magnetic field on the branes  $\hat{F}_{ij} = B_{ij}$ : this is the only consistent value for the background field strength in those directions allowed by tadpole cancellation. Since  $\hat{F}_{ij}$  is not integer is cannot be a field strength of a  $U(1)$  bundle but it must be defined in a non trivial bundle with a non abelian structure group on a torus. The string then describes the fluctuations around this background, these fluctuations have not the usual spectrum because of mechanism like a Scherk-Schwarz reduction and this explains in a very intuitive way why a configuration with  $B$  rank  $rk(B) = r$  reduces the rank of the gauge group as  $SO(2^{D/2}) \supset SO(2)^r \otimes SO(2^{D/2-r}) \rightarrow SO(2^{D/2-r})$ .

## References

- [1] M. Bianchi, G. Pradisi and A. Sagnotti, Nucl. Phys. B **376** (1992) 365.  
M. Bianchi and A. Sagnotti, “Twist symmetry and open string Wilson lines,” Nucl. Phys. B **361** (1991) 519.
- [2] M. Bianchi, “A note on toroidal compactifications of the type I superstring and other superstring vacuum configurations with 16 supercharges,” Nucl. Phys. B **528** (1998) 73 [arXiv:hep-th/9711201].
- [3] E. Witten, “Toroidal compactification without vector structure,” JHEP **9802** (1998) 006 [arXiv:hep-th/9712028].
- [4] C. Angelantonj, “Comments on open-string orbifolds with a non-vanishing  $B(ab)$ ,” Nucl. Phys. B **566** (2000) 126 [arXiv:hep-th/9908064].
- [5] C. Angelantonj and R. Blumenhagen, “Discrete deformations in type I vacua,” Phys. Lett. B **473** (2000) 86 [arXiv:hep-th/9911190].
- [6] C. Angelantonj and A. Sagnotti, “Open strings,” Phys. Rept. **371** (2002) 1 [Erratum-ibid. **376** (2003) 339] [arXiv:hep-th/0204089].
- [7] P. Di Vecchia, A. Liccardo, R. Marotta, I. Pesando and F. Pezzella, “Wrapped Magnetized Branes: Two Alternative Descriptions?,” JHEP **0711** (2007) 100 [arXiv:0709.4149 [hep-th]].

- [8] P. Di Vecchia, A. Liccardo, R. Marotta, F. Pezzella and I. Pesando, “Boundary state for magnetized D9 branes and one-loop calculation,” arXiv:hep-th/0601067.
- [9] I. Pesando, “Boundary states for branes with non trivial homology in constant closed and open background,” arXiv:hep-th/0505052.  
M. Billo, F. Denef, P. Fre, I. Pesando, W. Troost, A. Van Proeyen and D. Zanon, *Class. Quant. Grav.* **15** (1998) 2083 [arXiv:hep-th/9803228].
- [10] P. Di Vecchia, A. Liccardo, R. Marotta and F. Pezzella, “On the gauge / gravity correspondence and the open/closed string duality,” *Int. J. Mod. Phys. A* **20** (2005) 4699 [arXiv:hep-th/0503156].
- [11] D. Duo, R. Russo and S. Sciuto, *JHEP* **0712** (2007) 042 [arXiv:0709.1805 [hep-th]].